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It has been recently claimed that cosmologies with time dependent speed of light might solve some of the problems of the standard cosmological scenario, as well as inflationary scenarios. In this letter we show that most of this models, when analyzed in a consistent way, lead to large violations of charge conservation.

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Since one of the key hypothesis of special relativity is the frame independence of the velocity of light c, it is implied in this statement the time and space independence of this velocity. As well established that it may seem, this constancy principle has been recently contested [1] to provide an alternative account of the horizon, flatness and cosmological constant problems present in the standard big bang scenario. Instead of the superluminal expansion of the Universe present in inflationary scenarios, a period in which light traveled much faster than today would explain the homogeneity we see today in the Universe. Some cosmological models have also been analyzed afterwards [2,3] to test the dynamical viability of this scenario.

These ideas are highly provocative, not only from the observational viewpoint but also from the conceptual one. Indeed one of the key aspects of Einstein equivalence principle is the time-independence of the so called "fundamental constants" of physics [4]. The replacement of these parameters by one or more dynamical fields can lead to time- as well as space-dependent local fundamental constants. Unification schemes such as superstring theories [5] and Kaluza-Klein theories [6] have cosmological solutions in which the low-energy fundamental constants are functions of time. Usually low-energy phenomena are used to constrain the variation rate of the fundamental constants [7]- [18]. Moreover, non-null results have been announced recently [19]. If the cosmological dynamics of a field is such that its large-scale value is invariant under local Lorentz transformations, or if the local coupling with matter is strong enough so that it depends on the local environment (*e.g.* electromagnetism with the absorber condition), then the local field equations will be Lorentz invariant. If on the other hand the cosmological evolution is non-trivial and the field couples softly with the local matter, it will act as an external bath, breaking local Lorentz invariance. A variable speed of light theory may belong to the latter set of theories. Any VSL theory poses an additional problem, namely that c is a dimensional constant, and talking about a varying dimensional constant is not an invariant statement: we can change our units and obtain a different time dependence of such a parameter. Of course, once we fix our units, every claim about a dimensional parameter is an invariant claim, since we are implicitly referring to a dimensionless ratio: that between the parameter and the unit [15,16].

Any scientific theory has to be stated in clear and precise terms. Beckenstein's theory of a variable fine structure constant was based on Lorentz invariance, explicitly protecting charge conservation [20] (For versions of the theory that do not conserve charge, see [1]). In the case of VSL theories, local Lorentz invariance is relaxed, and the inhomogeneous Maxwell equations are assumed to be [1]

$$\frac{1}{c}\partial_\mu \left(cF^{\mu\nu}\right) = 4\pi j^\nu, \tag{1}$$

where $j^\mu = (\rho, \mathbf{j}/c)$ is the electric charge current. In reference [1] it was suggested that charge is conserved, implying a variation of the fine structure constant $\alpha = \frac{e^2}{\hbar c}$. The constancy of e can be derived, for instance, from Dirac's equation, written in Hamiltonian form:

$$i\hbar\partial_t\psi = -(i\hbar\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + \text{h.c.})\psi + mc^2\psi$$

which implies that $Q = e\int_V \psi^*\psi d^3x$ is conserved. The above form is, however, not unique since powers of c can be introduced in the equation in several ways, followed by appropriate symmetrization.

However, from equation (1) it can be easily seen that j^μ is no longer a conserved current, but satisfies the equation:

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$$4\pi\partial_\mu(cj^\mu) = \partial_\mu\partial_\nu(cF^{\mu\nu}) \quad (2)$$

The right hand side is not null, because the partial derivatives do not commute:

$$[\partial_0, \partial_i] = \frac{c_{,i}}{c^2}\partial_t$$

It is usually assumed that c is a function of a scalar field and that it depends only on time in the comoving cosmological frame. In the local frame of the solar system, moving with a velocity v with respect to the cosmological frame, a small space dependence will arise, with gradients $O(v/c)$ with respect to the time derivative, which can be neglected for the present purposes. So, the right hand side of equation 2 is effectively zero. The left hand side, however, is not a four divergence, because $\partial_{x^0} = 1/c(t)\partial_t$. The fully expanded expression is:

$$\partial_t\rho + \frac{\dot{c}}{c}\rho + \nabla\cdot\mathbf{j} = 0 \quad (3)$$

If equation (3) is integrated over a volume V containing the charges, we obtain

$$\frac{\dot{Q}}{Q} = -\frac{\dot{c}}{c} \quad (4)$$

where $Q = \int_V \rho d^3x$ is the total electric charge. This is our main result.

Equation (4) provides very stringent tests of the variation of c, since there have been many experiments to test the conservation of charge [21]. Depending on the details of the theory, several cases arise.

If we assume, as it is usually done in this context [1,2] that the electron charge e is constant, charge conservation can only be broken by processes that change charge discontinuously, such as the dissapearance of electrons or the transformation of neutrons into protons. Table I show some sample upper limits obtained from these processes with different techniques and hypothesis.

On the other hand, if e varies continuously in such a way that ce is conserved, then $\alpha \propto c^{-3}$ and strict limits can be obtained from geophysical or astronomical data, such as the Oklo phenomenon [13,14], the line spectra of distant quasars [12,19] or laboratory experiments [17].

These limits discard a great number of cosmological models with varying velocity of light. For instance, the family introduced in [2] parameterizes light velocity in the form: $c = c_0\left(\frac{a}{a_0}\right)^n$ with a the cosmological scale factor. It was shown in reference [2] that $n < -1/2$ is necessary to solve the flatness and horizon problems, and $n < -3/2$ solves the cosmological constant problem. In these models the following equation holds:

$$\frac{\dot{c}}{c} = n\frac{\dot{a}}{a} = nH_0 \quad (5)$$

From equation (5) we find the limits of table I on n, which contradict the above requirements. (We use $H_0 =$ 65 km/s/Mpc $= 6.65\times10^{-11}$ yr^{-1}). Similar bounds can be obtained other similar models, such as those studied in reference [25].

Finally, models similar to the original Albrecht-Magueijo one [1], involving a sudden change of c between two different constant values in the very early Universe, are not affected by the above limits. Orito and Yoshimura [26] observed that if charge conservation is broken in the very early Universe, a large charge excess should have been formed through a mechanism similar to that of baryogenesis: violation of Q, C and CP conservation while the system is out of thermodynamic equilibrium [27,28]. In the above mentioned models, the net charge excess will be produced by way-out-of-equilibrium production and decay of heavy mesons [28].

Let X be an unstable meson that produces a mean baryon number ϵ_B and a mean charge excess ϵ_Q per decay, and assume that matter is created during the charge transition period. Then, the equations for the evolution of the number densities of X, B, Q will be [28,2,1]:

$$\begin{aligned}(a^3n_X)^\bullet + \lambda_X(a^3n_X) &= \frac{3Kc\dot{c}}{4\pi Gm_X}a\\ (a^3n_B)^\bullet &= \epsilon_B\lambda_X n_X a^3\\ (a^3n_Q)^\bullet &= \epsilon_Q\lambda_X n_X a^3\end{aligned}$$

Following Albrecht-Magueijo [1], we assume that the change in c occurs in a short interval of time $t_c << \tau << \frac{1}{\lambda_x}$. Charge conservation will be broken only during this interval, but the X meson decay will always produce a baryon excess. With these hypothesis, the above equations have the following solutions:

$$
\begin{aligned}
a^3 n_X &\simeq \tfrac{3K(c_0^2-c_P^2)}{8\pi G m_X} a(0) e^{-\lambda_X t} &&= a^3(0) n_X^0 e^{-\lambda_X t} \\
a^3 n_B &\simeq \epsilon_B a^3(0) n_X^0 \left(1 - e^{-\lambda_X t}\right) &&\to_{t\to\infty} \epsilon_B a^3(0) n_X^0 \\
a^3 n_Q &\simeq \epsilon_Q \lambda_X \tau a^3(0) n_X^0 &&\simeq \frac{\epsilon_Q}{\epsilon_B} \lambda_X \tau a^3 n_B
\end{aligned}
$$

After the transition, n_Q will be fixed but n_B will be diluted from the above estimate by thermal processes [28]. So we finally get a lower bound on the charge excess:

$$\left|\frac{n_Q}{n_B}\right| \sim \frac{\epsilon_Q}{\epsilon_B} \lambda_X \tau \tag{6}$$

As we have mentioned before, we expect on general grounds that $\tau > t_{Pl}$, while $\epsilon_Q \sim \epsilon_B$, since these fractions depend both on the C and CP breaking terms in the lagrangean. Thus, equation (6) predicts a firm lower limit for the charge excess. Orito and Yoshimura [26] have given limits on any charge excess in the Universe, shown in table I. These limits are many orders of magnitude below the prediction of equation (6). The last column of the table shows rough estimates of τ taken from the observational limits, assuming $1/\lambda_X \sim t_{GUT}$.

Although these results do not rule out all varying velocity of light theories, they put very stringent bounds on them through the conservation of charge requirement. Moreover, similar bounds will hold for any theory with varying speed of light velocity in the early universe. These bounds may be lowered through improvements in the experimental techniques [29], and will lead into even tighter constraints on these interesting theories.

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Process	Ref.	Datum	$\mid \dot{Q}/Q \mid$ (y^{-1})	Param.
		τ (y)		$\mid n \mid$
$^{71}\mathrm{Ga} \to ^{71}\mathrm{Ga}$	[22]	$\geq 3.5 \times 10^{26}$	$\leq 2.9 \times 10^{-27}$	$< 5 \times 10^{-17}$
$e \to \nu_e \gamma$	[23]	$\geq 2.4 \times 10^{25}$	$\leq 4.2 \times 10^{-26}$	$< 7 \times 10^{-16}$
$e \to$ any	[24]	$\geq 2.7 \times 10^{23}$	$\leq 3.7 \times 10^{-24}$	$< 6 \times 10^{-14}$
		$\mid \Delta\alpha/\alpha \mid$		
Oklo phenomenon	[14]	$\leq 1.2 \times 10^{-7}$	$\leq 2.0 \times 10^{-16}$	$< 3 \times 10^{-6}$
Quasar absorption systems	[12]	$\leq 3.5 \times 10^{-4}$	$\leq 1.18 \times 10^{-13}$	$< 2 \times 10^{-3}$
Quasar absorption systems	[19]	$\leq 1.4 \times 10^{-5}$	$\leq 6.48 \times 10^{-15}$	$< 10^{-4}$
Laboratory constraint	[17]	$\leq 1.42 \times 10^{-14}$	$\leq 3.7 \times 10^{-14}$	$< 5.6 \times 10^{-4}$
		n_Q/n_B		τ/t_{Pl}
Coulomb force smaller than Newton force in stars	[26]	$< 10^{-18}$	—	$< 10^{-7}$
CMB anisotropy	[26]	$< 2 \times 10^{-20}$	—	$< 2 \times 10^{-9}$
Cosmic ray isotropy	[26]	$< 10^{-29}$	—	$< 10^{-18}$

TABLE I. Upper limits on charge non-conservation. The columns show the process considered, the corresponding references, the observational data, the charge non-conservation upper bound and the limits for the model parameters.

[1] A. Albrecht and J. Magueijo, *Phys. Rev. D* **59** 043516 (1999)
[2] J. D. Barrow, *Phys. Rev. D* **59** 043515 (1999)
[3] R. H. Brandenberger and J. Magueijo, hep-ph/9912247 (1999)
[4] C. M. Will *Theory and experiment in gravitational physics* 2nd edition. (Cambridge University Press, Cambridge, 1993)
[5] J. H. Schwartz *Phys. Rep.* **89** 223 (1982); M. B. Green and J. Schwartz *Phys. Lett.* **199B** 117 (1984)
[6] T. Kaluza *Sitz. Preuss. Akad. Wiss. Phys. Math.* **K11** 966 (1921); O. Klein *Z. Phys.* **371** 895 (1926)
[7] J. Ekkis, E. Gabathuler and M. Karliner *Phys. Lett. B* **217** 173 (1989); Riazuddin and Fayyazuddin *Phys. Rev. D* **81** 944 (1988)
[8] M. W. McElhinny, S. R. Taylor and D. J. Stevenson *Nature* **271** 316 (1978)
[9] T. Damour, G. W. Gibbons and J. H. Taylor *Phys. Rev. Lett.* **61** 1151 (1988)
[10] R. H. Hellings, P. J. Adams, J. D. Anderson, M. S. Keesey, E. L. Lau and E. M. Standish *Phys. Rev. Lett.* **51** 1609 (1983)
[11] A. M. Wolfe, R. L. Brown and M. S. Roberts *Phys. Rev. Lett.* **37** 179 (1976)
[12] L. L. Cowie and A. Songaila *Ap.J* **453** 596 (1995)
[13] A. I. Shlyakhter *Nature* **264** 340 (1976)
[14] T. Damour and F. Dyson *Nucl. Phys. B* **480** 37 (1996)
[15] P. Sisterna and H. Vucetich *Phys. Rev. D* **41** 1034 (1990)
[16] P. Sisterna and H. Vucetich *Phys. Rev. D* **44** 3096 (1991)
[17] J. D. Prestage, R. L. Tjoelker and L. Maleki, *Phys. Rev. Lett.* **74** 3511 (1995)
[18] J. B. Kim, J. H. Kim and H. K. Lee *Phys. Rev D* **58** 027301 (1998)
[19] J. K. Webb, V. V. Flambaum, C. W. Churchill, M. J. Drinkwater and J. D. Barrow *Phys. Rev. Lett.* **82** 884 (1999)
[20] J. Beckenstein *Phys. Rev. D* **25** 1527 (1982)
[21] L. B. Okun in *Review of particle properties Phys. Rev D* **45** Part II, VI.10 (1992)
[22] E. B. Norman, J. N. Bahcall and M. Goldhaber *Phys. Rev. D* **53** 4086 (1996)
[23] F. T. Avignone et. al. *Phys. Rev. D* **34** 97 (1986)
[24] D. Reusser et. al. *Phys. Lett. B* **353** 168 (1995)
[25] J. D. Barrow and J. Magueijo, astro-ph/9901049.
[26] S. Orito and M. Yoshimura *Phys. Rev. Lett.* **54** 2457 (1985)
[27] E. W. Kolb and M. S. Turner *Ann. Rev. Nuc. Part. Sci.* **33** 645 (1983)
[28] E. W. Kolb and M. S. Turner *The Early Universe* (Addison-Wesley, Redwood City, CA, 1990)
[29] R. Alonso, A. López García and H. Vucetich *Nucl. Instr. Meth. A* **383** 451 (1996)